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CIS 315: Intermediate Algorithms

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Assignment 1

1. An adjacency matrix A, that contains a vertex with in-degree n-1 and out-degree 0 is sometimes called a universal sink. A matrix can at most contain only one universal sink where a single row has all entries 0 and its corresponding column number has all entries 1. Therefore, an algorithm to determine such matrix in O(n) time of a directed graph is as follows:

Given the set of all vertices V, pick two unique vertices x,y∈V so that if A(x,y) = 1 then remove x from V, otherwise remove y. By doing so we are removing n-1 vertices that do not satisfy our condition. With one vertex, u, we must then check that its row is all 0’s and the column is 1. If no u exists, then there does not exists a universal sink.

1. Algorithm in O(n+r) time is as follows:

Represent each unique butterfly as a vertex with an edge representing a difference in species. Therefore, the graph with contain n vertices and r edges. Perform a BFS on a vertex v, to visit all the vertices and for each unique vertex u, if the distance from v to u is even then label it as group A, if it’s odd then label it group B. Finally, check that each edge exists between group A and B. This algorithm takes: O(n) time to assign each butterfly to a group, O(r) time to check every edge, and O(n + r) time to perform a BFS. Therefore, the overall run time of this algorithm is O(n + r).

1. Algorithm in linear time is as follows:

Given a directed graph G, with the n courses (vertices), we know that the minimum number of semesters is equivalent to the number of a prerequisites required for a particular course. We can represent each course as a vertex that is connected by an edge r, to represent the prerequisites of the previous class. Therefore, if we performed a DFS, the longest path will yield the minimum number of semesters needed. Thus, the overall run time of this algorithm is O(n + r).